Effective Gravitation Theory at Large Scale with Lorentz Violation

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Outline

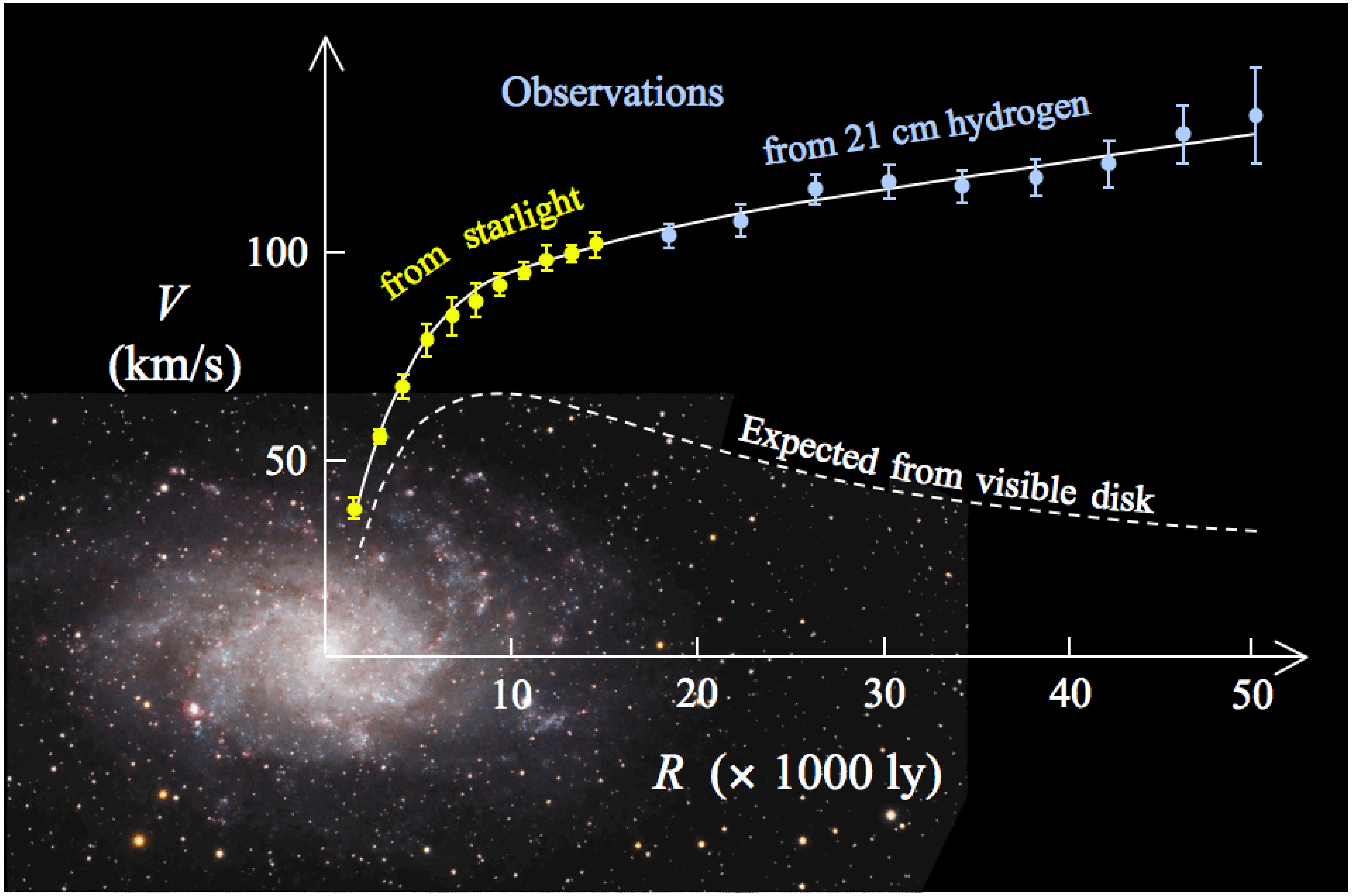
- 1) Introduction
- 2) Anistropies of CMB
- 3) Lorentz violation and VSR
- 4) Large scale effective gravity
- 5) Gauge Principle approach for Gravity
- 6) Possible extension
- 7) Conclusions-Outlooks

Deviation from GR

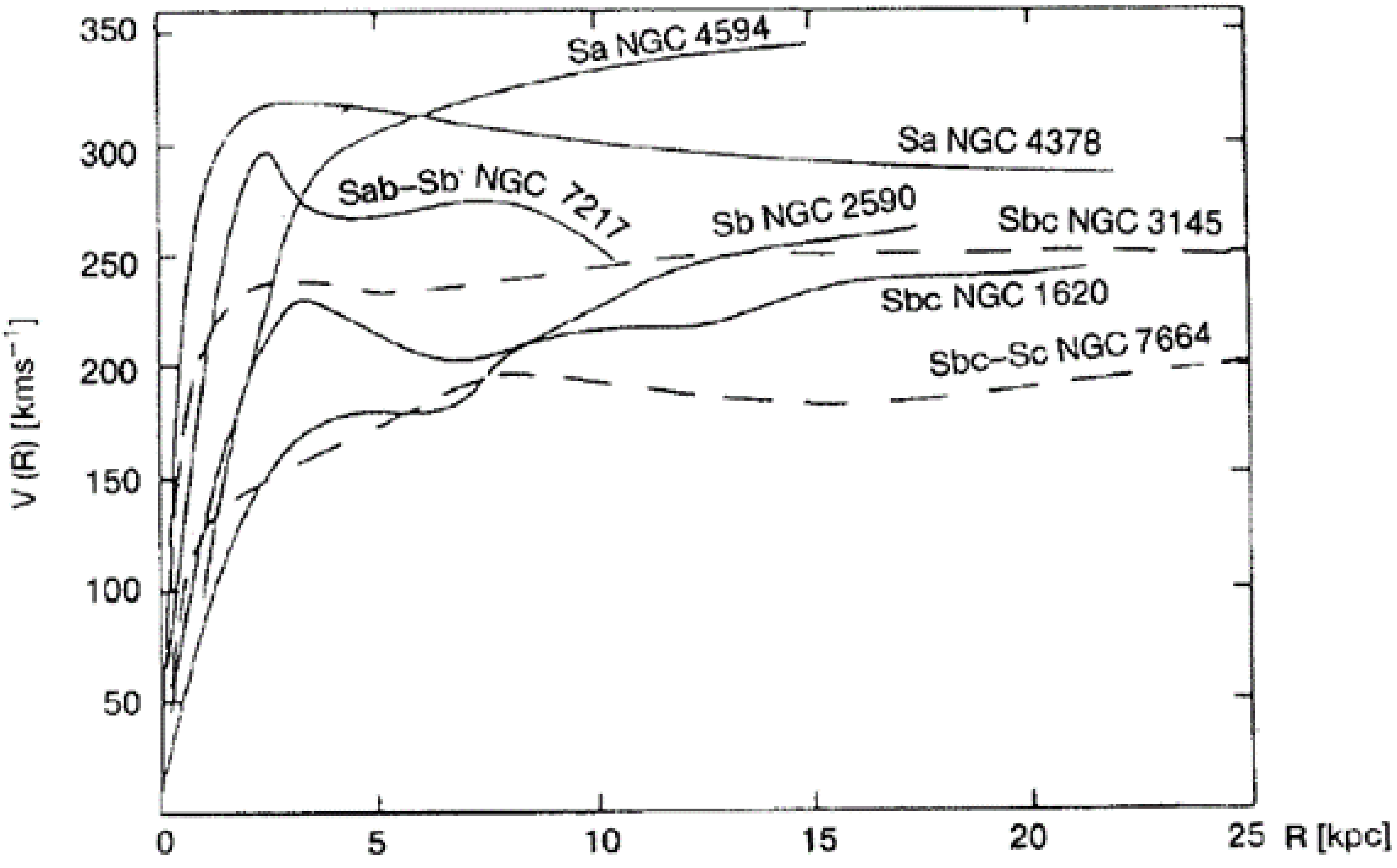
- Galaxy rotation curve etc.
- Acceleratiing expansion of the universe

V. Rubin; Ford (1970)

Galaxy rotation curve

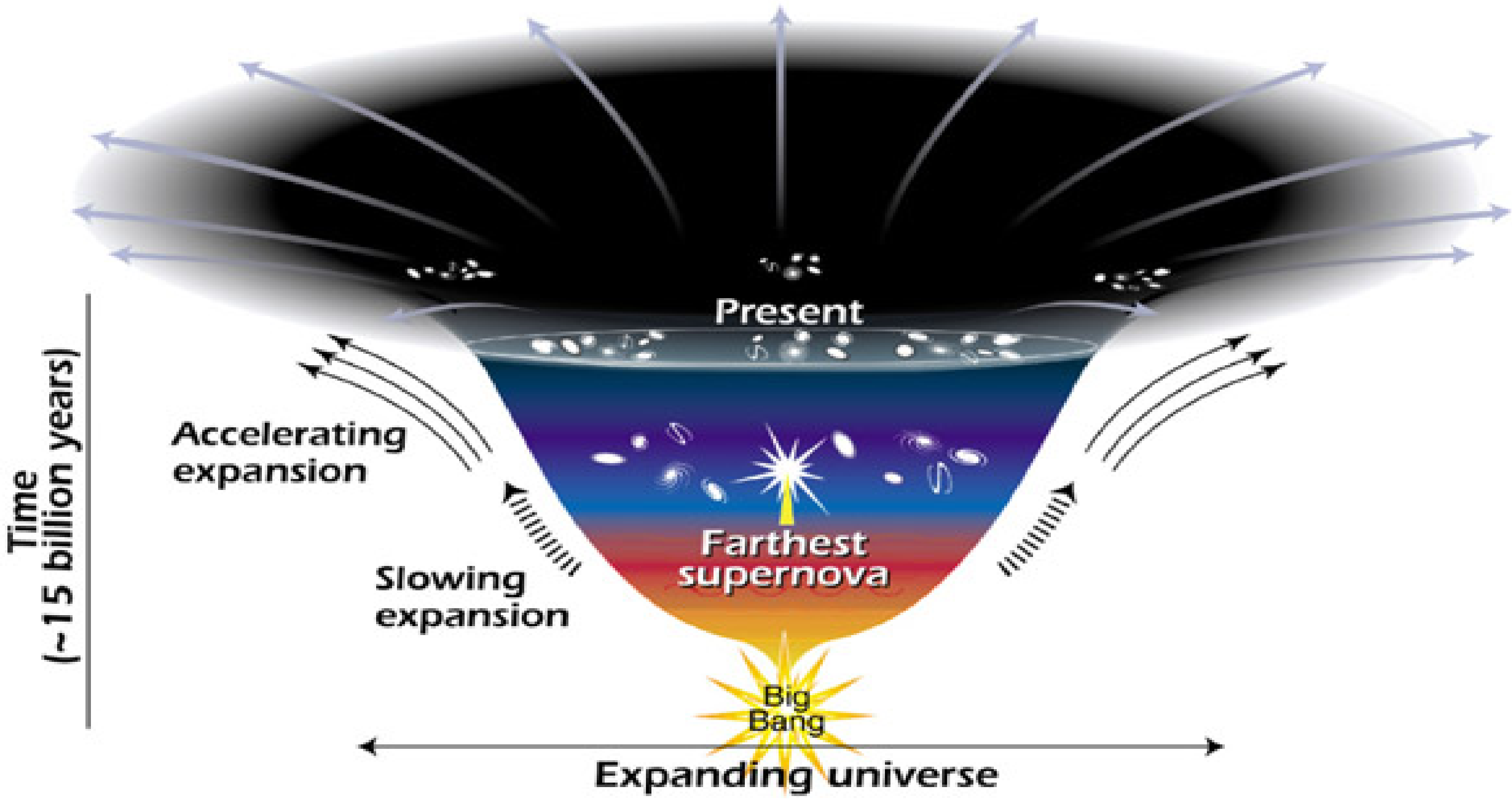


The dark Halo



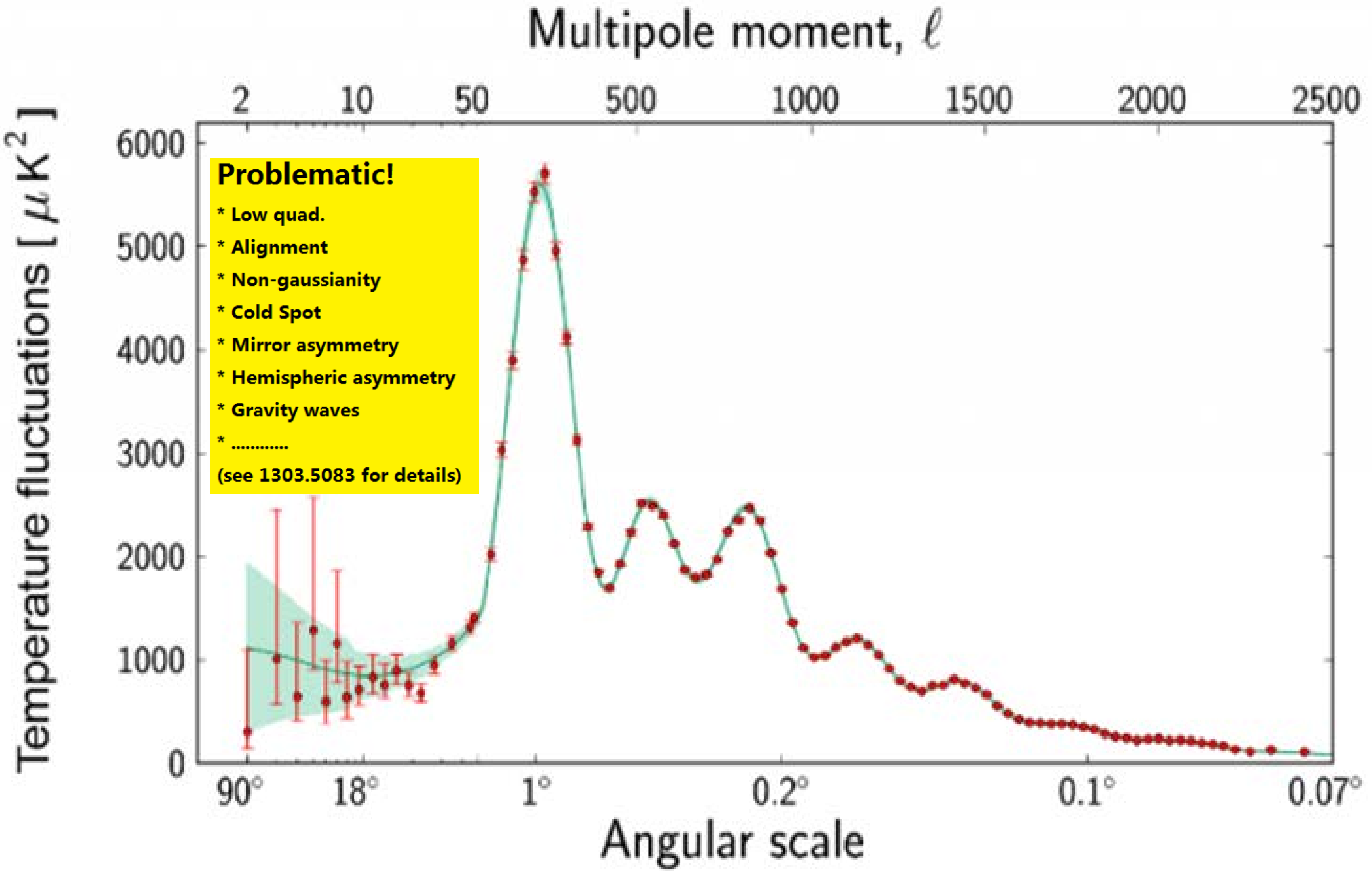
7 spiral galaxies. The flatness indicates the presence of huge dark halos. (V.J. Martinez, astro-ph/0203377).

Dark Energy and the Expansion of the Universe



This diagram reveals changes in the rate of expansion since the universe's birth 15 billion years ago. The more shallow the curve, the faster the rate of expansion. The curve changes noticeably about 7.5 billion years ago, when objects in the universe began flying apart at a faster rate. Astronomers theorize that the faster expansion rate is due to a mysterious, dark force that is pushing galaxies apart.

Anistropies of CMB



CMBR dipole anisotropy

- the existence of a CMB rest frame breaks Lorentz invariance even in empty space far away from any galaxy.
- alternative cosmological models can only explain some fraction of the observed dipole temperature distribution in the CMB.

Phys Rev D, 1999, 59, 116008

arXiv:1303.5087

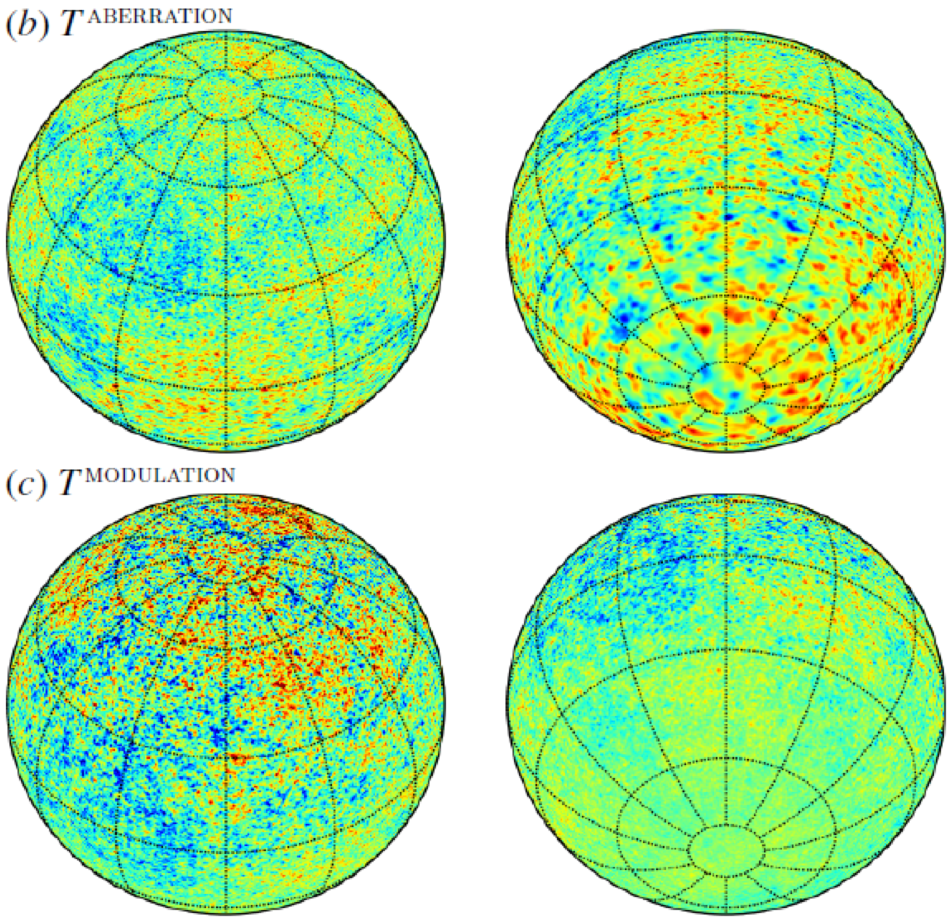


Fig. 1. Exaggerated illustration of the aberration and Doppler modulation effects, in orthographic projection, for a velocity $v = 260\,000\,\mathrm{km\,s^{-1}} = 0.85c$ (approximately 700 times larger than the expected magnitude) toward the northern pole (indicated by meridians in the upper half of each image on the left). The aberration component of the effect shifts the apparent position of fluctuations toward the velocity direction, while the modulation component enhances the fluctuations in the velocity direction and suppresses them in the anti-velocity direction.

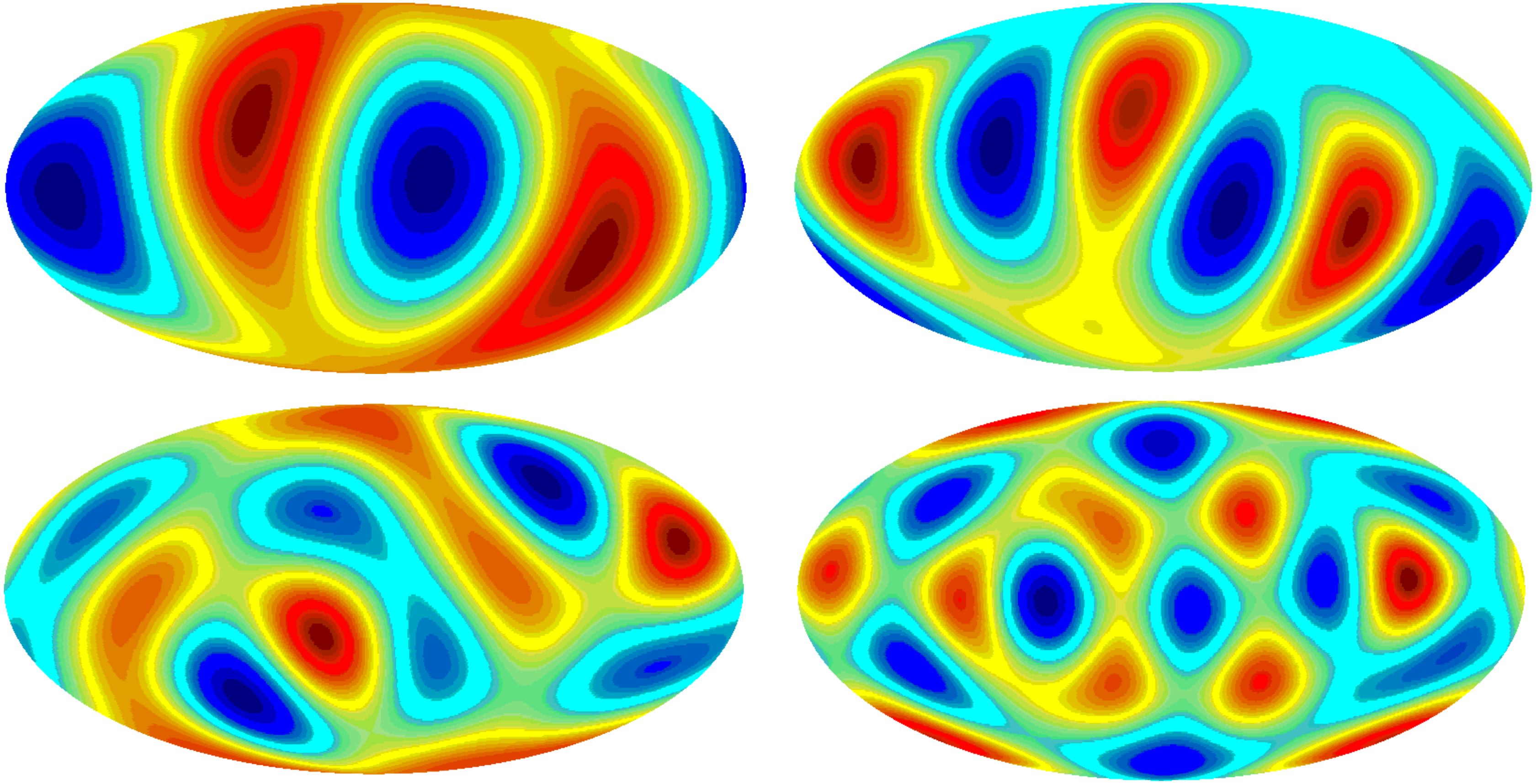
Robertson–Walker metric

- Based on the cosmological principle,the spacetime of the universe can be described by the Robertson–Walker metric at the cosmological scale.

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2 d\Omega\right]$$

- The contradiction with relativity principle

CMB Low Multipoles
------alignment of multipoles: l=2-5



Low multipoles and other anomalies

- CMB exhibits anomalies, such as very large scale anisotropies, anomalous alignments, and non-Gaussian distributions.
- the quadrupole (l = 2, spherical harmonic) has a low amplitude compared to the predictions of the Big Bang.
- the quadrupole and octupole (l = 3) modes appear to have an unexplained alignment with each other and with both the ecliptic plane and equinoxesan alignment sometimes referred to as the axis of evil.

arXiv:1506.07135

- Lorentz invariance might be lost apparently from the lenth scale of galaxy to the cosmic scale.

Lorentz violation, the theoretical investigations

- lot of attentions on the theoretical investigation and experimental examination of Lorentz symmetry since the mid of 1990s.
- Coleman and Glashow, boost invariance violation in the rest frame of the cosmic background radiation
- Colladay and Kostelecky standard model extension incorporating Lorentz and CPT violation
- Cohen-Glashow's very special relativity (VSR)model

Phys Rev D, 1998, 58, 116002

Phys Rev Lett, 2006,97, 021601

The identified VSR subgroups up to isomorphism

Phys Rev Lett, 2006,97, 021601

- T(2) (2-dimensional translations) with generators $T_1 = K_x + J_y$ and $T_2 = K_y - J_x$, where J and K are the generators of rotations and boosts respectively
- E(2) (3-parameter Euclidean motion) with generators T_1 ; T_2 and J_z ,
- HOM(2) (3-parameter orientation preserving transformations) with generators T_1; T_2 and K_Z
- SIM(2) (4-parameter similitude group)with generators T1;T2; Jz and Kz

Large scale effective gravity

- At the cosmic scale, Lorentz boost is violated.
- Inside the solar system, local Lorentz invariance is verified to astonishing accuracy
- Assuming the local Lorentz symmetry begins to break down from the scale of Galaxy scale to cosmic scale.
- Large scale effective gravitation theory should be a theory with local Lorentz invariance broken down

Mach’s principle-the reference guide

- A very general statement of Mach's principle is “*Local physical laws are determined by the large-scale structure of the universe*”

How large of an area can be regarded as local?

- The local reqirement differs from different physical phenomenon.

- We take very special relativity symmetry Sim(2),Hom(2) and E(2) gauge theories as an example to illustrate the so called dark matter and dark energy effect may be emerged from the Lorentz violation effect

Equivalence principle, local Sim(2) symmetry

- local Sim(2) symmetry invariant theory, gravity, the local transformation constrained on Sim(2).

$$\psi \xrightarrow{x^\mu \to \Lambda^\mu{}_\nu x^\nu} U\left(\Lambda\left(x\right)\right)\psi,\ \Lambda\left(x\right) \in Sim\left(2\right)$$

- The localization of the Sim(2) symmetry: introducing gauge potential or connection

$$\mathscr{L}\left(\partial_\mu \psi, \cdots\right) \longrightarrow \mathscr{L}\left(\mathscr{D}_\mu \psi, \cdots\right)$$

$$\mathscr{D}_\mu = \partial_\mu - \frac{i}{2} A^{ab}{}_\mu S_{ab}$$

- the Lorentzian group generator S_{ab}

- The connection 1-form:

$$A_\mu = A^{10}{}_\mu S_{10} + A^{20}{}_\mu S_{20} + A^{30}{}_\mu S_{30} + A^{12}{}_\mu S_{12} + A^{23}{}_\mu S_{23} + A^{31}{}_\mu S_{31}$$

$$= \frac{1}{2}\left(A^{10}{}_\mu + A^{31}{}_\mu\right)T_1 + \frac{1}{2}\left(A^{20}{}_\mu - A^{23}{}_\mu\right)T_2 + A^{30}{}_\mu K_3 + A^{12}{}_\mu J_3$$

$$+ \frac{1}{2}\left(A^{10}{}_\mu - A^{31}{}_\mu\right)\left(S_{10} - S_{31}\right) + \frac{1}{2}\left(A^{20}{}_\mu + A^{23}{}_\mu\right)\left(S_{20} + S_{23}\right)$$

- Tetrad field $h^a{}_\mu$
- Relation with metric: $g_{\mu\nu} = \eta_{ab} h^a{}_\mu h^b{}_\nu$

$$\eta_{ab} = g_{\mu\nu} h_a{}^\mu h_b{}^\nu$$

- relation with linear connection

$$A^a{}_{b\mu} = h^a{}_\nu \partial_\mu h_b{}^\nu + h^a{}_\nu \Gamma^\nu{}_{\rho\mu} h_b{}^\rho \equiv h^a{}_\nu \nabla_\mu h_b{}^\nu$$

- tetrad basis $h_a = h_a{}^\mu \partial_\mu$

- commutation relation $\left[h_a, h_b\right] = f^c{}_{ab} h_c$
- the structure coefficients

$$f^c{}_{ab} = h_a{}^\mu h_b{}^\nu \left(h^c{}_{\mu,\nu} - h^c{}_{\nu,\mu}\right)$$

- The curvature and torsion play the role of field strength of connection and tetrad fields.

$$D_a = h_a{}^{\mu}\mathscr{D}_{\mu} = h_a{}^{\mu}\left(\partial_{\mu} - \frac{i}{2}A^{cd}{}_{\mu}S_{cd}\right)$$

$$\left[D_a, D_b\right] = T_{ab}{}^{p}D_p + \frac{i}{2}R_{ab}{}^{pq}S_{pq}$$

the dynamics with constrains

- the Sim(2) constrains

$$A^{10}{}_{\mu} - A^{31}{}_{\mu} = 0, \;\; A^{20}{}_{\mu} + A^{23}{}_{\mu} = 0$$

- The action：

$$S_E = \frac{1}{16\pi G}\int d^4xh\left(R^{ab}{}_{ab} + \lambda_1{}^{\mu}\left(A^{10}{}_{\mu} - A^{31}{}_{\mu}\right) + \lambda_2{}^{\mu}\left(A^{20}{}_{\mu} + A^{23}{}_{\mu}\right)\right)$$

$$h = \det\left(h^a{}_{\mu}\right)$$

- EOM of connections

$$\mathscr{D}_{\nu}\left(h\left(h_a{}^{\nu}h_b{}^{\mu} - h_a{}^{\mu}h_b{}^{\nu}\right)\right) = \lambda_1{}^{\mu}h\left(\delta_a{}^1\delta_b{}^0 - \delta_a{}^3\delta_b{}^1\right) + \lambda_2{}^{\mu}h\left(\delta_a{}^2\delta_b{}^0 - \delta_a{}^2\delta_b{}^3\right)$$

- The constrained EOM for connections

$$\lambda_1{}^{\mu} h = \mathscr{D}_{\nu}\left(h\left(h_1{}^{\nu} h_0{}^{\mu} - h_1{}^{\mu} h_0{}^{\nu}\right)\right) = -\mathscr{D}_{\nu}\left(h\left(h_3{}^{\nu} h_1{}^{\mu} - h_3{}^{\mu} h_1{}^{\nu}\right)\right)$$

$$\lambda_2{}^{\mu} h = \mathscr{D}_{\nu}\left(h\left(h_2{}^{\nu} h_0{}^{\mu} - h_2{}^{\mu} h_0{}^{\nu}\right)\right) = \mathscr{D}_{\nu}\left(h\left(h_2{}^{\nu} h_3{}^{\mu} - h_2{}^{\mu} h_3{}^{\nu}\right)\right)$$

- And

$$\mathscr{D}_{\nu}\left(h\left(h_3{}^{\nu} h_0{}^{\mu} - h_3{}^{\mu} h_0{}^{\nu}\right)\right) = 0$$

$$\mathscr{D}_{\nu}\left(h\left(h_1{}^{\nu} h_2{}^{\mu} - h_1{}^{\mu} h_2{}^{\nu}\right)\right) = 0$$

- Independent number of Eqn: $24 - 2 \times 4 = 16$

- Free components of connections : 8

- The local Lorentz symmetry case leads to GR

$$S_E = \frac{1}{16\pi G}\int d^4 x h R_{ab}{}^{ab}$$

- EOM of Lorentz connection

$$\mathcal{D}_\nu\left(h\left(h_a{}^\nu h_b{}^\mu - h_a{}^\mu h_b{}^\nu\right)\right) = 0$$

- Leads to Levi-Civita connection

$$\Gamma^\rho{}_{\nu\mu} = \frac{1}{2} g^{\rho\lambda}\left(g_{\mu\lambda,\nu} + g_{\nu\lambda,\mu} - g_{\mu\nu,\lambda}\right)$$

- the spin connection can be decomposed into

$$A^a{}_{bc} = \tilde{A}^a_{bc} + K^a{}_{bc}$$

- the torsion free connection in GR, $\tilde{A}^a_{bc}$

$$\tilde{A}^a_{bc} = \frac{1}{2}\left(f_b{}^a{}_c + f_c{}^a{}_b - f^a{}_{bc} \right)$$

- the contorsion

$$K^a{}_{bc} = \frac{1}{2}\left(T_b{}^a{}_c + T_c{}^a{}_b - T^a{}_{bc}\right)$$

- Recall the local Lorentz case, the EOM for connection reduced to constrain on connections and resulted in Levi-Civita ones.
- In local Sim(2) case, the contorsion has 8 independent components,

$$K^{10}{}_{0}, K^{10}{}_{1}, K^{10}{}_{2}, K^{20}{}_{0}, K^{30}{}_{0}, K^{30}{}_{1}, K^{30}{}_{2}, K^{12}{}_{0}$$

- The 8 constrain eqns reduced to:

$$f^{1}{}_{10} + f^{1}{}_{31} = 0 \qquad 2f^{0}{}_{20} - f^{2}{}_{30} + f^{3}{}_{20} - f^{0}{}_{23} = 0$$

$$f^{2}{}_{23} - f^{2}{}_{20} = 0 \qquad 2f^{0}{}_{10} + f^{0}{}_{31} - f^{1}{}_{30} + f^{3}{}_{10} = 0$$

$$f^{2}{}_{03} + f^{0}{}_{23} + 2f^{3}{}_{23} - f^{3}{}_{20} = 0 \qquad f^{2}{}_{10} + f^{2}{}_{31} + f^{0}{}_{12} + f^{3}{}_{12} + f^{1}{}_{20} - f^{1}{}_{23} = 0$$

$$f^{1}{}_{30} + f^{0}{}_{31} + f^{3}{}_{10} + 2f^{3}{}_{31} = 0 \qquad -f^{1}{}_{20} + f^{3}{}_{12} + f^{0}{}_{12} - f^{2}{}_{10} + f^{1}{}_{23} - f^{2}{}_{31} = 0$$

- EOM for the tetrad field: $$R_c{}^a - \frac{1}{2}\delta_c{}^a R = 0$$
- The decomposition of curvature

$$R^{mn}{}_{ab} = \tilde{R}^{mn}_{ab} + R_K{}^{mn}{}_{ab} + R_{CK}{}^{mn}{}_{ab}$$

- the curvature in GR $\tilde{R}^{mn}_{ab}$
- And $$R_K{}^{mn}{}_{ab} \equiv \left(h_a{}^\nu h_b{}^\mu - h_a{}^\mu h_b{}^\nu\right)\left(\partial_\nu K^{mn}{}_\mu + K^m{}_{e\nu} K^{en}{}_\mu\right)$$

$$R_{CK}{}^{mn}{}_{ab} \equiv \left(h_a{}^\nu h_b{}^\mu - h_a{}^\mu h_b{}^\nu\right)\left(\tilde{A}^m_{e\nu} K^{en}{}_\mu + K^m{}_{e\nu}\tilde{A}^{en}_\mu\right)$$

- The Einstein equation in local Lorentzian case

$$\tilde{R}_c{}^a - \frac{1}{2}\delta_c{}^a \tilde{R} = 0$$

- In local Sim(2) case

$$\tilde{R}_c{}^a - \frac{1}{2}\delta_c{}^a \tilde{R} = 8\pi G \left(T_{Sim(2)} \right)_c{}^a$$

- And

$$\left(T_{Sim(2)} \right)_c{}^a = \frac{1}{8\pi G}\left(\frac{1}{2}\delta_c{}^a \left(R_K + R_{CK} \right) - \left(R_{K\,c}{}^a + R_{CK\,c}{}^a \right) \right)$$

The dynamics with Local Sim(2)

- It can be viewed as the non-Einstein gravity part $T_{Sim(2)}$ may contribute effectively as dark matter.

- It should be noted that the non-Einstein gravity contribution $T_{Sim(2)}$

vanishes identically if the whole space is empty. The Minkowski space is still a solution of the equation.

Local Sim(2) , with presence of source matter field

- In the presence of source matter field

$$\delta S_M = \int d^4 x h \left(\frac{1}{2} \delta h^c{}_a \left(T_M\right)_c{}^a + \delta A^{ab}{}_\mu \left(C_M\right)_{ab}{}^\mu \right)$$

- the full EOM for tetrad field under constrain condition :

$$\tilde{R}_c{}^a - \frac{1}{2}\delta_c{}^a \tilde{R} = 8\pi G \left(T_{Sim(2)} + T_M \right)_c{}^a$$

$$A^{10}{}_\mu - A^{31}{}_\mu = 0, \;\; A^{20}{}_\mu + A^{23}{}_\mu = 0$$

- The connection satisfy eqns

$$\mathscr{D}_\nu\left(h\left(h_1{}^\nu h_0{}^\mu - h_1{}^\mu h_0{}^\nu\right)\right) + \mathscr{D}_\nu\left(h\left(h_3{}^\nu h_1{}^\mu - h_3{}^\mu h_1{}^\nu\right)\right) = 16\pi G\left[\left(C_M\right)_{10}{}^\mu + \left(C_M\right)_{31}{}^\mu\right]$$

$$\mathscr{D}_\nu\left(h\left(h_2{}^\nu h_0{}^\mu - h_2{}^\mu h_0{}^\nu\right)\right) - \mathscr{D}_\nu\left(h\left(h_2{}^\nu h_3{}^\mu - h_2{}^\mu h_3{}^\nu\right)\right) = 16\pi G\left[\left(C_M\right)_{20}{}^\mu - \left(C_M\right)_{23}{}^\mu\right]$$

$$\mathscr{D}_\nu\left(h\left(h_3{}^\nu h_0{}^\mu - h_3{}^\mu h_0{}^\nu\right)\right) = 16\pi G\left(C_M\right)_{30}{}^\mu$$

$$\mathscr{D}_\nu\left(h\left(h_1{}^\nu h_2{}^\mu - h_1{}^\mu h_2{}^\nu\right)\right) = 16\pi G\left(C_M\right)_{12}{}^\mu$$

- In GR, scalar source field implies zero torsion while spinor does not. Local Sim(2) theory has a spin connection with torsion in general even in the scalar field source case. And there is a energy-momentum source contributed by contorsion in addition to the energy- momentum source contributed from matter.

The Self Consistency of Sim(2) Gauge Theory

- Employing the constrain 8 eqns

$$A^{10}{}_{\mu} - A^{31}{}_{\mu} = 0, \;\; A^{20}{}_{\mu} + A^{23}{}_{\mu} = 0$$

- obtain the Sim(2) invariant theory, we make the substitution

$$\partial_{\mu} \to \mathscr{D}_{\mu} = \partial_{\mu} - i\left(A^{10}{}_{\mu}T_1 + A^{20}{}_{\mu}T_2 + A^{12}{}_{\mu}J_3 + A^{20}{}_{\mu}K_3\right)$$

- One need to verify the Maurer–Cartan eq. holds within Sim(2) algebra and the Bianchi Identity holds for the curvature and torsion of the Sim(2) connection.
- The curvature 2-form is indeed closed within the Sim(2) algebra

$$R^{pq}{}_{ab}S_{pq}=2\left(h_a{}^{\nu}h_b{}^{\mu}-h_a{}^{\mu}h_b{}^{\nu}\right)$$

$$\Big[\left(\partial_\mu A^{10}{}_\nu+A^{12}{}_\mu A^{20}{}_\nu-A^{10}{}_\mu A^{30}{}_\nu\right)T_1$$

$$+\left(\partial_\mu A^{20}{}_\nu-A^{12}{}_\mu A^{10}{}_\nu-A^{20}{}_\mu A^{30}{}_\nu\right)T_2$$

$$+\partial_\mu A^{12}{}_\nu J_3+\partial_\mu A^{30}{}_\nu K_3\Big]$$

- With the contribution from torsion, one gets the Maurer–Cartan eq. on sim(2) algebra

$$\left[\mathcal{D}_a,\mathcal{D}_b\right]=T^p{}_{ab}\mathcal{D}_p+\frac{i}{2}R^{pq}{}_{ab}S_{pq}$$

- By the Jacobi Identity

$$\left[\mathcal{D}_m,\left[\mathcal{D}_n,\mathcal{D}_p\right]\right]+\left[\mathcal{D}_p,\left[\mathcal{D}_m,\mathcal{D}_n\right]\right]+\left[\mathcal{D}_n,\left[\mathcal{D}_p,\mathcal{D}_m\right]\right]=0$$

- and Maurer–Cartan eq. and the Sim(2) constrain, the first Bianchi Identity

$$\mathcal{D}_d T^a{}_{bc} + \mathcal{D}_c T^a{}_{db} + \mathcal{D}_b T^a{}_{cd}$$

$$= R^a{}_{bcd} + R^a{}_{dbc} + R^a{}_{cdb}$$

$$+ T^e{}_{bd} T^a{}_{ec} + T^e{}_{dc} T^a{}_{eb} + T^e{}_{cb} T^a{}_{ed}$$

- and the second Bianchi Identity still hold.

$$\mathcal{D}_\nu R^a{}_{b\rho\mu} + \mathcal{D}_\mu R^a{}_{b\nu\rho} + \mathcal{D}_\rho R^a{}_{b\mu\nu} = 0$$

- Similar analysis for other VSR subgroups.

Extensions to Local Hom(2), E(2) and T(2) are Straitforward

The Finsler geometry realization of VSR

- Gibbons, Gomis and Pope : possible quantum corrections or the quantum gravity effect, ISIM(2) admits a 2-parameter family of continuous deformations, keeping space-time remains flat
- only a 1-parameter DISIMb(2) is physically acceptable.
- The line element invariant under DISIMb(2) is Lorentz violating and of Finsler type, $ds^2 = (\eta_{\mu\nu} dx^\mu dx^\nu)^{1-b} (n_\mu dx^\mu)^{2b}$.

The Perturbative Solution of the Representation of the Deformation group Generators

- The natural representation of the deformed generators is the representation inherit from the Poincar´e group's 5 dimensional natural matrix representation
- We get all of possible natural representation of the deformed Poincar´e subgroups we obtained.
- There may be more than one set of solutions, which corresponding to different spacetime geometry.

SCI CN Phy, Mech & Astro，57 (2014)859-874
arXiv:1205.11346

Summary of The Invariant Metric

- The invariant metric function for deformed Poincare subgroup

$$F^2 = (A_\mu y^\mu)^{2-2\sum_{a,b} D_{a,b}} \prod_{a,b} \left(B_{(a,b)\mu\upsilon} y^\mu y^\upsilon\right)^{D_{a,b}}.$$

where A_μ can be N_μ, T_μ and X_μ while $B_{(a,b)\mu\upsilon}$ can take $\tilde{G}_{(a,b)\mu\upsilon}$, $B_{(a,b)\mu\upsilon}$ and $H_{(a,b)\mu\upsilon}$. For different groups, the metric usually are: $F^2 = G_{\mu\upsilon} y^\mu y^\upsilon$, $(N_\mu y^\mu)^2$ or $(G_{\mu\upsilon} y^\mu y^\upsilon)^{1-A} (N_\mu y^\mu)^{2A}$.

- Invariant vectors A_μ, N_μ, T_μ and X_μ, invariant tensor: $B_{\mu\nu}$, $G_{\mu\nu}, H_{\mu\nu}$

DISIMb(2) symmetry

- Take DISIMb(2) as an example, the Finslerian line element can be any dimension 2 function of $\eta_{\mu\nu}dx^{\mu}dx^{\nu}$ and $\left(n_{\mu}dx^{\mu}\right)^{2}$.
- DISIMb(2)

$$[T_1, J_z] = -T_2 \quad [T_1, K_z] = T_1 \quad [T_2, J_z] = T_1 \quad [T_2, K_z] = T_2$$

$$[T_1, P_t] = P_x \quad [T_1, P_x] = P_t - P_z \quad [T_1, P_z] = P_x$$

$$[T_2, P_t] = P_y \quad [T_2, P_y] = P_t - P_z \quad [T_2, p_z] = P_y$$

$$[J_z, P_x] = P_y \quad [J_z, P_y] = -P_x$$

$$[K_z, P_t] = P_z + bP_t \quad [K_z, p_z] = P_t + bP_z$$

$$[K_z, P_x] = bP_x \quad [K_z, p_y] = bP_y$$

Local DISIMb(2) symmetry

- The gauge theory with Local DISIMb(2) symmetry is in progress. One may expect such kind of theory describes Lorentz violating gravitation with a continuous parameter.

Teleparallel Gravity

- It is well known that Teleparallel Gravity is an equivalent approach to gravity to GR.
- However, it need to be investigated how to define a Lorentz violated gravity within Teleparallel Gravity framework.

Conclusions

- 1) At cosmic scale boost invariance is implicitly broken, however, at scale larger than the galaxy, the boost invariance may not be broken totally.
- 2) The very special relativity symmetry, is illustrated as an example of large scale local symmetry and its corresponding gravity theory is constructed.
- 3) All VSR gauge theories are gravity theory with non-trivial torsion in general. At least part of dark matter effects might be emerged from contribution by contortion.
- 4)Dark matter effect might be an emerge effect from large scale Lorentz violation.

Outlook

- 1) The gauge gravity construction of Finslerian spacetime based on the deformation of VSR symmetries.
- 2)The galaxy rotation curve
- 3) The cosmological application: RW metric solution and the possible effective dark energy emerged from contribution by contortion
- 4) The relation with other Lorentz violating gravity such as massive gravity (Zhakaharov discontinuity and the solution) and the nonlocal approach by Deser and Woodard

THANKS!